The quadratic B-spline curve fitting for the shape of log cross sections

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Abstract: This paper describes a new method for simulation of the cross section shape of log. The self-developed MQK3102 log shape recognizing machine was used to acquire the finite discrete sampling points on the cross section of log and those points were fitted with the quadratic B-spline parametric curve. This method can clearly stimulate the real shape of the log cross section and is characterized by limited sampling points and high speed computing. The computed result of the previous curve does not affect the next one, which may avoid the graphic distortion caused by the accumulative error. The method can be used to simulate the whole body shape of log approximately by sampling the cross sections along the length direction of log, thus providing a reference model for optimum saw cutting of log.

Keywords: Quadratic B-spline curve; Cross section shape; Log; Computer simulation; Optimum saw cutting

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Introduction

The contour characteristic of log is extremely complex. To recognize the log shape by traditional dimensional measuring method may produce errors inevitably between theoretical saw cutting and the actual cutting (Tong *et al*. 1997). With further studies on the model of log shape as well as the extensive use of computer graphics processing technology in recent years, it has been possible to simulate accurately the outline of log on computer (Gong 2001). These theories and methods are vitally significant to the saw cutting optimization (Ma *et al*. 2005) and the directional cutting of fibre (Ma 2005). The key for simulating the outline of log is to establish a proper mathematic model to make the fitted graph conform farthest to the actual contour characteristics of log. In this study, we adopted the quadratic B-spline parametric curve to fit the cross section shape of log, so as to provide software and model for the numerical control processing of directional cutting of fibre.

Acquisition of sampling points on log cross section

We acquired the sampling points on the cross section of log by using the self-developed MQK3102 log shape recognition machine. Eight detecting pendulum bars are arranged in circle on the detecting frame at the center section of the machine. A roller with very short diameter is installed at the bottom end of each pendulum bar to contact the surface of log, and an angle transducer is installed on the mandrel at the fixed end of each pendulum bar. When the log moves to the detecting frame, one of its ends pushes the eight pendulum bars, and then the eight contact rollers fall onto the log. The fluctuation on the surface of log causes the pendulum bars to swing, thus leading to the change of the output values from the angle sensor.

Fig. 1 and Fig. 2 show the abridged general view of the rectangular detecting frame. The O is an origin of world coordinate system in Figs..The horizontal direction along its bottom margin is the axis X, and the vertical direction along its left margin is the axis Y. Here we suppose that D_x and D_y are respectively the horizontal ordinate and the vertical ordinate of the pendulum bar's fixed end. d represents the projection of the pendulum bar to the detecting frame plane XOY. α is the intersect angle between the pendulum bar and the negative direction of axis X. Therefore, the coordinate value of the sampling points on the cross section of log can be defined by Equation (1) as follows:

$$\begin{cases} x = D_x + d\cos\alpha \\ \mathrm{y} = D_y - d\sin\alpha \end{cases} \tag{1}$$

Suppose that the length of each pendulum bar is l, and its tilt angle is θ, then the projection d can be obtained from the Equation (2):

$$d = l\cos\theta \tag{2}$$

In Equation (1) and Equation (2), D_x , D_y , α and l are determined by the designing variables of MQK3102 log shape recognition machine, and θ_i is acquired real-timely by each angular transducer.

Establishment of mathematic model

B-spline curve is the improved Bezier curve. As to the Bezier curve, changing one of the control points will affect the shape of curve completely, which is not in favor of the selective modification to curve. Moreover, in most cases the curve approximated by Bezier can not reflect the features of the characteristic polygon (Sun 2000; Sun *et al*. 1995).

The parametric equation of quadratic B-spline curve (Sun 2000) is:

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$$P(t)=\sum_{i=0}^{2}P_iG_{i,2}(t)=\sum_{i=0}^{2}P_i\left(\frac{1}{2!}\sum_{j=0}^{2-i}(-1)^j C_{2+1}^{j}(t+2-i-j)^2\right)$$
$$0\le t\le 1 \tag{3}$$

The expanded form is expressed with matrix as follows:

$$P(t)=\frac{1}{2}\begin{pmatrix}t^2 & t & 1\end{pmatrix}\begin{bmatrix}1 & -2 & 1\\ -2 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\end{bmatrix}\quad 0\le t\le 1 \tag{4}$$

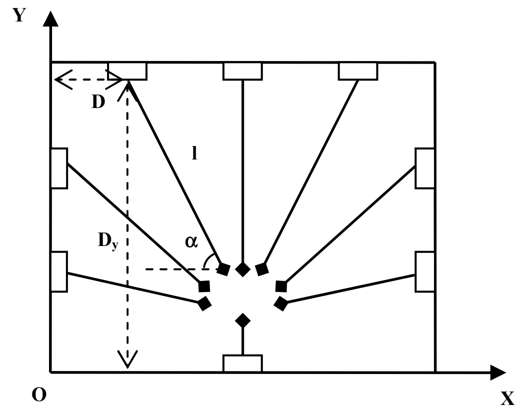


Fig.1 Front view of detecting frame

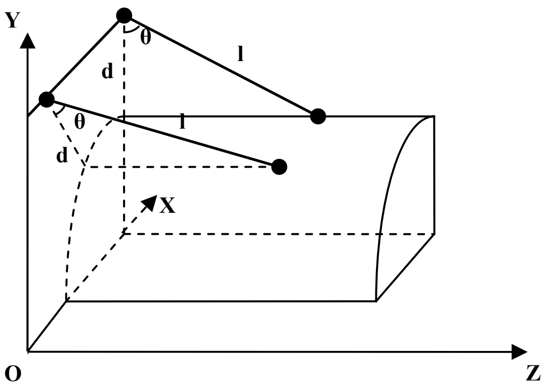


Fig.2 Side view of detecting frame

The three vectors P_0, P_1 and P_2 are resolved respectively into $P_i(x_i, y_i)$ which is in X and Y directions on two-dimensional space, then the component form is expressed as Equation (5):

$$\begin{cases} x(t)=\frac{1}{2}\begin{pmatrix}t^2 & t & 1\end{pmatrix}\begin{bmatrix}1 & -2 & 1\\ -2 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_0\\ x_1\\ x_2\end{bmatrix}\\ y(t)=\frac{1}{2}\begin{pmatrix}t^2 & t & 1\end{pmatrix}\begin{bmatrix}1 & -2 & 1\\ -2 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}y_0\\ y_1\\ y_2\end{bmatrix}\end{cases} \tag{5}$$

After expanding equation (5), the parametric equation of quadratic B-spline curve is obtained according to the descending powers of parameter t.

$$\begin{cases} x(t)=(\frac{1}{2}x_0-x_1+\frac{1}{2}x_2)t^2+(x_1-x_0)t+\frac{1}{2}(x_0+x_1)\\ y(t)=(\frac{1}{2}y_0-y_1+\frac{1}{2}y_2)t^2+(y_1-y_0)t+\frac{1}{2}(y_0+y_1)\end{cases} \tag{6}$$

From Equation (4) we can get:

$$\begin{cases} P(0)=\frac{1}{2}(P_0+P_1)\\ P(1)=\frac{1}{2}(P_1+P_2)\end{cases} \tag{7}$$

Then differentiate the function $P(t)$ to first order:

$$P'(t)=\frac{1}{2}\begin{pmatrix}2t & 1 & 0\end{pmatrix}\begin{bmatrix}1 & -2 & 1\\ -2 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\end{bmatrix}\quad 0\le t\le 1 \tag{8}$$

And from equation (8) we can get the results below:

$$\begin{cases} P'(0)=P_1-P_0\\ P'(1)=P_2-P_1\end{cases} \tag{9}$$

Therefore, we can conclude from Eqs. (7) and (9) that the start point and the end point of curve P is the midpoint of P_0P_1 and P_1P_2 respectively. Moreover, curve P is tangent with P_0P_1 and P_1P_2, just like what is shown by Fig.3.

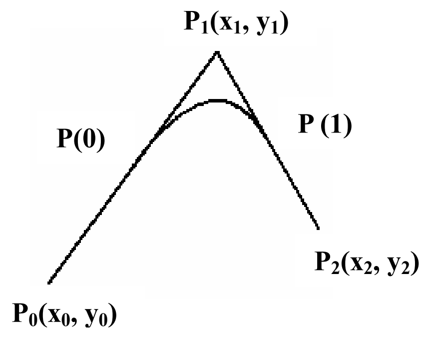


Fig.3 Quadratic B-spline curve determined by three points

From what is mentioned above, we can deduce that to the quadratic B-spline curve controlled by n $(n>3)$ points, every two neighboring curve segments are tangent at their junction point, which satisfies the conditions of both $P_i(1)=P_{i+1}(0)$

and $P_i'(1) = P_{i+1}'(0)$ at one time. Thus, the quadratic B-spline curve has fine continuity and smoothness.

Realizing the Curve Fitting for the Outline of Cross Section of Log

According to equation (6), the parametric equation of the *i*th quadratic B-spline curve is obtained. It is arranged as follows:

$$\begin{cases} x_i(t) = [(\frac{1}{2}x_{i-1} - x_i + \frac{1}{2}x_{i+1})\,t + (x_i - x_{i-1})]\,t + \frac{1}{2}(x_{i-1} + x_i) \\ y_i(t) = [(\frac{1}{2}y_{i-1} - y_i + \frac{1}{2}y_{i+1})\,t + (y_i - y_{i-1})]\,t + \frac{1}{2}(y_{i-1} + y_i) \end{cases}$$

$$i = 1, 2, \cdots \qquad (10)$$

Supposed that the eight sampling points acquired from MQK3102 log shape recognition machine are $p_0, p_1 \cdots p_7$ respectively, which are used as the control points during the process of quadratic B-spline curve fitting. The parameter $t\ (0 \le t \le 1)$ is divided into 100 equidistant intervals, that is $\triangle$t=0.01. After that we can work out the positions of the fitting points according to equation (10) when t is valued differently. And then join these fitting points end to end, so the smooth segment C_i is decided with the three sampling points $p_{i-1}\ p_i\ p_{i+1}$. When i is valued from 1 to 6, we can obtain six smooth segments of quadratic B-spline curve, and every two neighboring smooth segments of quadratic B-spline curve are tangent at the junction.

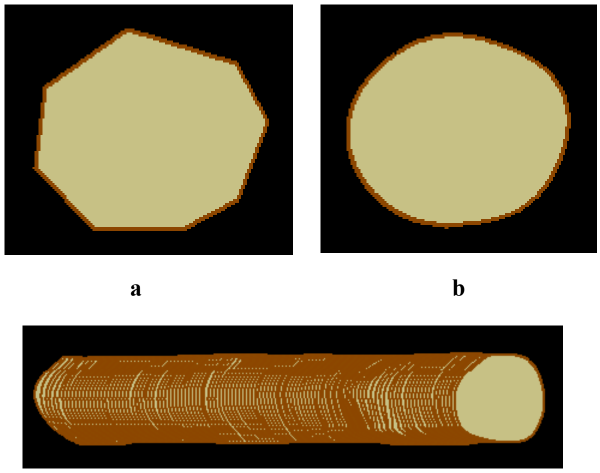
In addition, because of the shape of the cross section of log is the closed, we add two other control points p_8 and p_9, and also make them meet the conditions of $p_8 = p_0$ and $p_9 = p_1$ .That is, we can determine the smooth segments C_7 and C_8 which decided by $p_6 p_7 p_0$ and $p_7 p_0 p_1$ respectively according to the method mentioned above. By now, we have realized the curve fitting for the outline of cross section of log. Here we take one of the cross sections of the log for example. The coordinate of eight sampling points along the circumference are (290, 365), (210, 301), (200, 222), (270, 159), (347, 160), (400, 192), (432, 267) and (408, 326) (The unit is millimeter). So after processing in the computer, the fitting results are showed by Fig.4.

Conclusion

We use the self-developed MQK3102 log shape recognizing machine to acquire the finite discrete sampling points on the cross section of log and fit those points by the quadratic B-spline curve. Such a method can clearly stimulate the real shape of the log cross section, does not need many sampling points, and the computing speed is high.

The computed result of the previous curve does not affect the next one, which may avoid the graphic distortion caused by the accumulative error (Tong *et al*. 1996).

As is shown by Fig. 4 (c), if we collects certain cross sections in the length direction on the log, and follow the way mentioned above, the three-dimensional outline of log may be simulated approximately. The closer the sample interval is, the better the simulating effect will be.



a b

c

Fig. 4 Fitting results

a. The shape of cross section before curve fitting; b. The shape of cross section after curve fitting; C. The effect of simulation after getting certain cross sections in the length direction on the log

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